This is the Dirac equation with $c = 1$ and $\hbar = 1$.

$$i\left(\gamma^0\frac{\partial}{\partial t}+\gamma^1\frac{\partial}{\partial x}+\gamma^2\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial z}\right)\psi = m\psi$$

The following gamma matrices are the "Dirac representation."

$$\gamma^0=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad\gamma^1=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\quad\gamma^2=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\quad\gamma^3=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$$

The wave function ψ has angular frequency ω equal to the energy of the particle.

$$\omega=\sqrt{k_x^2+k_y^2+k_z^2+m^2}$$

There are four positive frequency solutions that are linearly independent.

$$\psi_1=\begin{pmatrix}\omega+m\\0\\k_z\\k_x+ik_y\end{pmatrix}\exp[i(k_xx+k_yy+k_zz-\omega t)]\quad\psi_2=\begin{pmatrix}0\\\omega+m\\k_x-ik_y\\-k_z\end{pmatrix}\exp[i(k_xx+k_yy+k_zz-\omega t)]$$

$$\psi_3=\begin{pmatrix}k_z\\k_x+ik_y\\\omega-m\\0\end{pmatrix}\exp[i(k_xx+k_yy+k_zz-\omega t)]\quad\psi_4=\begin{pmatrix}k_x-ik_y\\-k_z\\0\\\omega-m\end{pmatrix}\exp[i(k_xx+k_yy+k_zz-\omega t)]$$

There are four negative frequency solutions that are linearly independent. The negative frequency solutions flip the sign of m.

$$\psi_5=\begin{pmatrix}\omega-m\\0\\k_z\\k_x+ik_y\end{pmatrix}\exp[-i(k_xx+k_yy+k_zz-\omega t)]\quad\psi_6=\begin{pmatrix}0\\\omega-m\\k_x-ik_y\\-k_z\end{pmatrix}\exp[-i(k_xx+k_yy+k_zz-\omega t)]$$

$$\psi_7=\begin{pmatrix}k_z\\k_x+ik_y\\\omega+m\\0\end{pmatrix}\exp[-i(k_xx+k_yy+k_zz-\omega t)]\quad\psi_8=\begin{pmatrix}k_x-ik_y\\-k_z\\0\\\omega+m\end{pmatrix}\exp[-i(k_xx+k_yy+k_zz-\omega t)]$$

The following solutions are used by quantum electrodynamics.

ψ_1	Fermion, spin up
ψ_2	Fermion, spin down
ψ_7	Anti-fermion, spin up
ψ_8	Anti-fermion, spin down

This script verifies all of the solutions.

```
-- www.eigenmath.org/dirac-equation.txt

-- Verify solutions to the Dirac equation.

gamma0 = ((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma1 = ((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))
gamma2 = ((0,0,0,-i),(0,0,i,0),(0,i,0,0),(-i,0,0,0))
gamma3 = ((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))

omega = sqrt(kx^2 + ky^2 + kz^2 + m^2)

psi1 = (omega + m, 0, kz, kx + i ky) exp(i (kx x + ky y + kz z - omega t))
psi2 = (0, omega + m, kx - i ky, -kz) exp(i (kx x + ky y + kz z - omega t))
psi3 = (kz, kx + i ky, omega - m, 0) exp(i (kx x + ky y + kz z - omega t))
psi4 = (kx - i ky, -kz, 0, omega - m) exp(i (kx x + ky y + kz z - omega t))
psi5 = (omega - m, 0, kz, kx + i ky) exp(-i (kx x + ky y + kz z - omega t))
psi6 = (0, omega - m, kx - i ky, -kz) exp(-i (kx x + ky y + kz z - omega t))
psi7 = (kz, kx + i ky, omega + m, 0) exp(-i (kx x + ky y + kz z - omega t))
psi8 = (kx - i ky, -kz, 0, omega + m) exp(-i (kx x + ky y + kz z - omega t))

D(psi) = dot(gamma0,d(psi,t)) +
         dot(gamma1,d(psi,x)) +
         dot(gamma2,d(psi,y)) +
         dot(gamma3,d(psi,z))

"checking wave functions (1=ok)"

i D(psi1) == m psi1
i D(psi2) == m psi2
i D(psi3) == m psi3
i D(psi4) == m psi4
i D(psi5) == m psi5
i D(psi6) == m psi6
i D(psi7) == m psi7
i D(psi8) == m psi8
```